Capital Allocation, the Leverage Ratio Requirement and Banks' Risk-Taking[*]

Ioana Neamțu[†]

Bank of England

Quynh-Anh Vo[‡]

Bank of England

January 31, 2021

Abstract

This paper examines how the level at which regulatory constraints are applied affects banks' asset risk. We develop a theoretical model and calibrate it to UK banks. Our main finding is that the impact differs depending on which of regulatory constraints are binding at the group consolidated level. As long as at the consolidated level, only the leverage ratio constraint is binding, the allocation of constraints down does not imply any negative impact on banks' resilience. However, it could bring about an increase in banks' asset risk in the case where only the risk-weighted constraint binds at the group level. We also find that the impact on banks' asset risk of the application level differ across banks' business models.

JEL Classification: G21, G28.

Keywords: Leverage ratio requirement, risk-weighted capital requirements, capital allocation.

---

1 Introduction

Following the 2007 – 2009 financial crisis, the global regulatory capital framework has undergone substantial reform to address shortcomings in the pre-crisis framework and deliver a resilient banking system that can support the real economy. One of the prominent changes is the introduction of a leverage ratio requirement to complement the risk-based capital requirements and thus, to make the whole regulatory capital framework more robust to uncertainties about the true riskiness of banks' assets. This introduction however brings about significant challenges for banks to efficiently manage their limited financial resources. These challenges include how to incorporate multiple regulatory constraints within their capital allocation framework. Banks are therefore developing new approaches to allocating capital to their businesses. The evolution of banks' practices in turn may have implications for the effectiveness of regulatory measures. One important question, as highlighted in Bajaj et al. (2018), that requires further analysis is how the banks' riskiness depends on the way they treat the leverage ratio requirement within their capital allocation process.

This paper aims to shed light on this question. We examine how banks' asset risk is affected by the level (i.e. group or business unit level) at which regulatory constraints are applied. To do that, we construct a two-period model where there is a bank that runs two business units. One has higher non-risk adjusted returns but is also riskier, while the other has lower returns and is less risky. We model the two business units as a lending business and a repo business respectively. The bank is subject to two main regulatory constraints: the risk-weighted (RW) capital requirements, formulated using the Value-at-Risk (VaR) constraint, and the leverage ratio (LR) requirement. Those constraints are legally applied at the bank's group consolidated level but the bank can choose to require each of their business units to comply with both constraints - hereafter referred to as the bank allocating the two constraints to their businesses. We then compare how the bank's optimal investments will change if it chooses to do so. Our focus is on the level of risk of the bank's assets which is measured in our setup by the fraction of the bank's total balance sheet invested in the lending business.

One of the key insight we get is that the impact differs depending on which of the two

constraints are binding at the group consolidated level. We first consider the situation where the average risk weight (ARW) of the banking group is lower than the ratio between the minimum LR requirement and the minimum RW requirement, which implies that the bank is constrained by the LR at the group level. In that case, we characterise two situations in which changing the application level of regulatory constraints from the group level to the business unit level either does not change the bank's asset risk or lead to its decrease.

First, if the ARW of both business units on a stand-alone basis are also below the ratio of the two minima, then applying regulatory constraints at the business unit level does not change investment decisions of the LR-constrained bank as compared to group-level application. That is because in this situation the optimal investment at the group level already allows two business units to comply with the relevant constraint when regulatory constraints are allocated down.

On the other hand, if the stand-alone ARW of both business units are higher than the ratio of two minima, then applying both regulatory constraints at the business-unit level will lead to a decrease in the LR-constrained bank's asset risk. The intuition lines in the difference of the marginal cost of reallocating investment from repo business to lending business between applying constraints at the group level and allocating them down to business units. In the former case, since the binding constraint is the LR, that marginal cost is determined by the difference between the leverage cost per unit of lending and the leverage cost per unit of repo. In the latter case, the binding constraints are the RW requirements, which implies that the marginal cost is now determined by the difference between the marginal RW capital cost of lending business and that of repo business. Given that repo business has higher leverage cost but lower marginal RW capital cost than lending business, moving one unit of investment from repo business to lending business will lead to an increase of required capital resources in the case of applying constraints down to business unit level but to a decrease in those resources when constraints are applied at the group level. Therefore the bank will prefer to invest relatively more in the lending business in the latter case than in the former one. Put differently, the bank's asset risk is lower when regulatory

constraints are applied at the business unit level.

The allocation of constraints could bring about an increase in the bank's asset risk if at the group level, the risk-weighted constraint is the only binding constraint. Conditions for this result to happen include the fact that the riskier business unit is bound by the risk-weighted capital requirement, while the less risky business unit is bound by the leverage ratio on a stand-alone basis. Two other conditions relate to the comparison between the leverage cost of repo business and its marginal RW capital cost and the characteristic of the marginal diversification benefits. Together, those conditions can make the marginal cost of reallocating investment from repo business to lending business lower when applying constraints at business unit level than when constraints are imposed at the group level. This in turn makes the reallocation of investment from repo to lending beneficial for the banks when constraints are applied at the business unit level and the bank will prefer to invest relatively more in the lending business in that case. Therefore, the bank's asset risk increases as compared to the case where constraints are applied at the group level.

Given that it is not possible to analytically characterise all possible changes in the bank's investments following the allocation of regulatory constraints to business units, we complement our analytical insights with the numerical simulations. We calibrate the model using data on large UK banks. In our numerical results, we indeed find that the allocation of constraints leads to an increase of asset risk of the average bank in our sample when at the group level the bank is bound by the risk-weighted constraint only.

To understand the role of the business model, we examine whether the impact of the allocation of the constraints differ across banks with different business models. For this purpose, we first classify the UK banks in our sample into two types of banks, namely retails banks and wholesale and capital market-oriented banks. The average risk weights for retail activity are lower than average risk weights associated with wholesale and capital market activities. Hence retail banks are more likely to be bounded by leverage ratio, while wholesale and capital market-oriented are more likely to be constrained by the risk-weighted assets requirement. We recalibrate the model to each type of bank and run the

numerical simulations. The most interesting finding is that there is a stark difference in the impact of the allocation of constraints on the asset risk between the two bank types. This is especially the case where only the risk-weighted constraint matters at the group level, while the allocation of constraints results in an increase in asset risk of retail banks, it leads to a decrease in the asset risk of wholesale and capital market-oriented banks.

The organisation of the paper is as follows. After discussing the related literature in the next section, we set out in Section 3 our theoretical setup. Section 4 presents our main analytical insights. Then in Section 5 we calibrate our model to the UK banks and explain our numerical simulations in Section 6. In Section 7 we further break down our UK bank sample into the two types of banks outlined above, and discuss our new simulation results in this context. Finally Section 8 concludes and suggests further research.

2 Related Literature

This research adds to three strands of existing literature. We add to the literature on capital allocation under constraints, where we assess the impact of leverage ratio on internal capital allocation between different business units. Secondly, our research is related to the broader literature on unintended consequences of post-crisis financial regulation, more specifically on the leverage ratio requirement impact on bank risk-taking. Lastly, we apply our theoretical model to the repo and lending market, and substantiate with our theory the observed decrease in repo transactions since the introduction of leverage ratio.

The literature on the allocation of capital within complex financial institutions is still very limited. Capital allocation is a method used to determine the notional amount of equity capital needed to support a business. The performance of a business unit, and its respective capital allocation can be assessed and done in a large variety of ways.[1] An important factor in capital allocation among different business units is the marginal profitability relative to the costs of capital associated with each business unit. The allocation depends on the

---

[1]For a recent primer on capital allocation methods implemented in practice see Ita (2017).

relative profitability of each business unit in relation to the minimum marginal amount of capital required to perform operations. Among the most common measures of relative profitability are the Risk Adjusted Returns on Capital (RAROC) and Economic Value Added (EVA) which relate to economic capital, or measures that take into account the regulatory capital requirements (Ita, 2017). Economic capital is a measure of calculating a minimum amount of equity needed to cover unexpected shortfalls measured via Value-at-Risk (VaR) or Expected Shortfall (ES), and regulatory capital allocation relates to the minimum Basel III requirements on equity to risk-weighted assets, and more recently to the leverage ratio requirements.[2] An Oliver Wyman report finds that the minimum capital requirements started playing a much more central role in banks' capital allocation after the financial crisis of 2008 (Khaykin et al., 2017). The reason behind this is that the banks attach the highest weight to the most binding constraint of capital requirements, and regulatory capital took that role compared to other measures such as economic capital when regulatory requirements were tightened following the crisis.

A main criticism brought to different methods of capital allocation relates to calculating the cost of capital at an aggregate value of the firm, and not per project. In early work on the topic, Stein (1997) provides a rationale for the establishment of internal capital markets, which enable firms to shift resources across business units according to their relative profitability. But Graham and Harvey (2001) find that banks traditionally use aggregate risk, via the weighted average cost of capital (WACC), when evaluating projects, and not project-specific risk. This finding is supported empirically by Krüger et al. (2015), which argue that the aggregate allocation type leads to under-investment in safer businesses, and over-investment in relatively riskier ones. Another finding which connects costs of capital and risk profiles is made by Baker and Wurgler (2015), which argue that an increase in regulation that makes banks less risky lead to an increase in cost of capital, and they substantiate their hypothesis by an analysis of US bank returns. Moreover, Perold (2005) shows

---

[2]For more methodological details, Dhaene et al. (2012) analyse the existing literature and practice on different capital allocation methods and their rationale, and construct a unified capital allocation framework, which encompasses a large majority of these cases.

that accounting for diversification benefits between different units can reduce banks' capital needs. He suggests that banks should evaluate business activities based on their marginal contribution to expected operating profits and to the banks required risk capital.

In our paper the minimum amount of capital required depends on the leverage ratio and the risk-sensitive regulatory requirement, as this measure of required capital is the most commonly used measure at the time when this paper is written. Additionally, we take into account the criticism of using the average capital charge, by assessing both the cases when the regulatory capital constraints are applied at group level and at separate business unit levels. For instance, the low-risk, low-margin activities have a low risk-based capital charge, but then the risk insensitive leverage ratio has a relatively higher capital charge compared to the risk-based one. This effect is amplified for low margin and high balance sheet intensity activities such as repo transactions. In contrast, the high-risk, high-margin businesses derive the highest capital charge from risk-based requirements, while the leverage ratio requirement is less likely to be binding. This might be the case for lending activity.

In that sense, the most relevant paper related to our work is Goel et al. (2019) which analyses the allocation of capital across two business units (i.e. lending and market making) by banks that face multiple constraints, namely risk-based capital and leverage ratio requirements. They develop a theoretical model and calibrate it to US data, and show how shocks to one unit lead to spillover effects to other units in terms of allocated capital. Although our model is in similar spirit, Goel et al. (2019) formally analyse the impact of tightening constraints on capital allocation and business unit investments, while we focus on the risk-taking implications of applying the same constraints at different levels of a bank.

The impact of LR requirement on the banking portfolio composition is a relatively new research topic. Blum (2008) is the first paper to advocate for introducing this risk-insensitive measure to provide incentives for truthful risk reporting in an adverse selection model. The Basel Committee provides three reasons for the introduction of the leverage ratio requirement: help against excessive leverage build up, help against banks trying to circumvent the risk-based capital requirements and lastly, help against model risk (Basel Committee, 2009).

In more recent work, both Kiema and Jokivuolle (2014) and Acosta-Smith et al. (2018) find in theoretical setups that the leverage ratio introduction leads to an increase in risk-taking under certain conditions. Kiema and Jokivuolle (2014) focus on the model risk argument, and find that the shift in risk-taking does not affect the aggregate risk profile and banking stability, as banks re-shuffle the loans: banks focused on low-risk lending will shift towards more high-risk lending, while the high-risk lending banks will reallocate part of their portfolio to low-risk investments.Acosta-Smith et al. (2018) focus on the complementing risk-based capital requirements argument, and find an increase in risk-taking if equity is sufficiently costly, or banks are bound by the leverage ratio. They confirm these results empirically for a large panel of European banks. Choi et al. (2018) find similar empirical results for the US, where banks shift towards riskier investments (higher asset risk), but the shift is counterbalanced by increased capital leading to no change in overall bank risk. Our paper focuses on the asset risk implications of the leverage ratio introduction as a complement to risk-based capital requirements. Nonetheless unlike previous literature we do not assess how the introduction of LR requirement changes the portfolio composition within a business model, but rather how the allocation of the constraint at different business levels impacts asset risk. Other work assessing the leverage ratio requirement introduction focuses on business cycle and long-run interaction with risk-based requirements (Gambacorta and Karmakarb, 2018), or moral hazard and debt absorption capacity(Barth and Seckinger, 2018).

The most likely operations to be negatively affected by the leverage ratio requirement introduction are operations with a low expected return, but a high capital charge, such as repurchase agreements. Early evidence shows that the repo markets have been hit by the LR requirement both in Europe and the US (BIS CGFS, 2017; Allahrakha et al., 2018; Duffie, 2018). Nonetheless, there is no clear evidence of causation or the persistency of the effect of the leverage requirement. In a recent study on market liquidity in the UK gilt, Bicu-Lieb et al. (2020) find that a decrease in the repo liquidity coincides with the introduction of the leverage ratio requirement, but there is no clear evidence on the existence of a causal relationship between the two. Kotidis and Van Horen (2019) also analyse the UK gilt repo

market and conclude that the introduction of the leverage ratio has no impact on bilateral repo transactions. They find that repo dealers constrained by the leverage ratio decrease initially the transacted volumes with smaller clients, but this effect was temporary. The repo dealers who were not affected by the LR requirement took over the smaller clients, and later on the affected dealers increased haircuts on reverse repo transactions, in order to pass through increased costs of regulation. We do not perform an empirical analysis akin the previous two studies, but rather we emulate some of the (UK gilt) repo characteristics in our theoretical model and simulation. We consider the two types of balance sheet charges that repo transactions have, and estimate UK gilt repo market returns, which we later use to simulate banks' investment decisions.

3 The model

We consider a bank that runs two business units, one yields higher non-risk adjusted returns but is riskier than the other. Although our results will hold for any combination of two businesses that have those characteristics, in this paper, we model the riskier business as a lending business and the safer business as a repo business.

Two business units   The lending unit grants loans to customers. We denote the bank's ex-ante gross interest income from loans by $G(L)$ where L is the total value of granted loans. We capture the fact that granting loans is a risky business by assuming that ex-post some borrowers default. Denote by $\tilde{Z}$ the random variable that represents the losses per unit of loans. Therefore, the bank's ex-post lending revenue is equal to $G(L) - ZL$ where Z is the realised value of $\tilde{Z}$. We assume that $\tilde{Z}$ is distributed according to the distribution H_Z, h_Z with expected value equal to μ_Z.

The repo unit owns a stock of government bonds of value X with coupon c. It uses this government bonds inventory to raise collateralised fundings to finance bond trading activities or to act as an intermediary entering into repo transaction with some counterparties and offsetting reverse repo with others. We assume that the ex-ante income from repo activities

is equal to $F(X)$. We capture the risk of the repo business by assuming that ex-post the bank could suffer losses $\tilde{\varepsilon}X$ due to, for example, unpaid payments by reverse repo counterparties or losses from trading activities. Distribution of $\tilde{\varepsilon}$ is characterised by $H_\varepsilon, h_\varepsilon$ with expected value μ_ε.

We make the following assumptions on the profitability and riskiness of the two business units.

Assumption 1. *Functions $G(.)$ and $F(.)$ satisfy the following conditions:*

$$G(0) = 0; \quad G'(.) > 0 \quad and \quad G''(.) < 0 \tag{1}$$

$$F(0) = 0; \quad F'(.) > 0 \quad and \quad F''(.) < 0 \tag{2}$$

Assumption 1 implies that both lending and repo businesses have diminishing marginal returns. For lending business, this property could be explained by the fact that the loan interest rate is a decreasing function of loan size. For repo business, this could be due to the fact that interest rate on reverse repo is less sensitive to the transactional amount than the repo rate.

Assumption 2. *The rank between the two functions $G(.)$ and $F(.)$ is as follows:*

$$G'(y) - \mu_Z > F'(y) + c - \mu_\varepsilon \quad for\ all \quad y \leq max(X^*, L^*) \tag{3}$$

where

$$X^* = \operatorname*{argmax}_y \left[F(y) + cy - \mu_\varepsilon y - Ry\right] \quad and \quad L^* = \operatorname*{argmax}_y \left[G(y) - \mu_Z y - Ry\right] \tag{4}$$

Assumption 2 indicates that lending business is more profitable than repo business on a non risk-adjusted basis. Note that X^* and L^* defined in this assumption represent the size of, respectively, the repo and lending businesses so that expected profits from those activities are maximised when their costs of funding is R. The banks will never grant more loans than L^* and never hold an inventory of government bonds of value higher than X^*.

10

Assumption 3. *Two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ are independently distributed and ranked as follows:*

$$VaR_{1-q}(\tilde{Z}) \geq VaR_{1-q}(\tilde{\varepsilon}) \tag{5}$$

where $VaR_{1-q}(Y)$ denotes the Value at Risk (VaR) of a random variable Y at confidence level $1-q$, which is defined as:

$$VaR_{1-q}(Y) \equiv \inf\{y : \mathbb{P}(Y \geq y) \leq q\} \tag{6}$$

Assumption 3 states a ranking between two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ based on the VaR measure. It implies that lending business is riskier on a stand-alone basis than repos business.

Funding structure and capital allocation   We assume that the bank finances its assets with equity of amount K and debt with gross interest rate R. Among the total capital resources K the bank has, K_L will be allocated to support lending business while K_X is allocated to the repo business.

Figure 1 illustrates the balance sheet of the bank at the consolidated level and at the business-unit level. It is worth noting that the size of the balance sheet of the repo business is recorded as a multiple α of X to capture different possible regulatory treatment of repo activities. For example, when the bank runs a matched repo book, if all reverse repo transactions are not eligible to netting, due to the requirement that securities sold as collateral cannot be removed from the bank's balance sheet, the size of the repo business will be equal to $2X$.

We denote by $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ the profit of, respectively, the lending and repo business units. Therefore, $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ can be written as follows:

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L) \tag{7}$$

and

Figure 1: Bank's balance sheet

$$\tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X) \tag{8}$$

The overall profit of the bank at the consolidated level is thus equal to $\tilde{\Pi}_L + \tilde{\Pi}_X$

Regulatory constraints   The bank is subject to the two regulatory constraints, namely the leverage ratio (LR) requirement and the risk-weighted capital requirement. In line with the principle underlying the Basel requirements, we formulate the risk-weighted capital requirement using the VaR constraint.

Given the bank's balance sheet as described in Figure 1, the two regulatory constraints can be written at the consolidated level as follows:

$$K \geq \chi(L + \alpha X) \tag{9}$$

where χ is the required minimum LR and

$$\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a \tag{10}$$

In words, Constraint (10) states that the probability of total losses of the bank's asset

portfolio being higher then its capital is lower than a. After some algebra, it can be rewritten as[3]:

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L, X)}{R} \tag{11}$$

where

$$\Pi(L, X) = \underbrace{G(L) - RL}_{\equiv \Pi_L(L)} + \underbrace{F(X) + cX - RX}_{\equiv \Pi_X(X)}$$

In the Basel III framework, the right hand side (RHS) of Constraint (11) is equivalent to the product of the minimum risk-weighted (RW) capital requirements and the risk-weighted assets (RWAs) of the bank at the group level. We denote the former by γ and the latter by RWA^G. RWA^G can thus be proxied in our model by:

$$RWA^G = \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L, X)}{\gamma R}$$

If both business units have to comply with both constraints individually, then the lending business has to satisfy:

$$\mathbb{P}\left(\tilde{\Pi}_L \leq 0\right) \leq a \quad \text{and} \quad K_L \geq \chi L$$

while the repo business has to satisfy:

$$\mathbb{P}\left(\tilde{\Pi}_X \leq 0\right) \leq a \quad \text{and} \quad K_X \geq \chi \alpha X$$

The two individual VaR constraints can similarly be expressed in terms of RWA^L and RWA^X - the RWAs of, respectively, the lending and repo businesses on a stand-alone basis - as follows:

$$K_L \geq \gamma RWA^L \quad \text{where} \quad RWA^L = \frac{VaR_{1-a}(\tilde{Z}L) - \Pi_L(L)}{\gamma R} \tag{12}$$

---

[3]See Appendix A.1 for detailed derivation

and

$$K_X \geq \gamma RWA^X \quad \text{where} \quad RWA^X = \frac{VaR_{1-a}(\tilde{\varepsilon}X) - \Pi_X(X)}{\gamma R} \tag{13}$$

4 Analysis

We now analyse the bank's optimal investments. Our main objective is to investigate how the bank's asset risk is affected by the level at which the two regulatory constraints are applied. To do so, we will compare the bank's optimal investments in each business unit between the case where the two constraints are imposed at the group level and the case in which both business units have to individually comply with both regulatory constraints. In the following, we first formulate the bank's problem for each of these two scenarios. Then, we examine how the bank's investment decision differs between them.

4.1 Bank's optimisation problems

Optimisation problem with constraints applied at the group's level When all constraints are applied at the group level, the bank's optimisation problem, denoted as $\wp^G$, can be written as follows:

$$Problem \quad \wp^G: \quad \underset{L,X}{\text{Max}} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (9) and (11).

Our focus is on how the level at which the two regulatory constraints are applied affects the bank's asset risk. Given that the bank runs two businesses with one riskier than the other, the bank's asset risk in our model can be measured by the fraction of the bank's total balance sheet devoted to lending business - the riskier one. We denote by w this fraction, i.e.

$$w = \frac{L}{L + X} \tag{14}$$

14

The bigger w is, the higher the bank's asset risk. To facilitate the examination of how w would change depending on the application level of the two regulatory constraints, we reformulate Problem $\wp^G$ by changing the bank's decision variables from (L, X) to w and the bank's total balance sheet size $S = L + X$. After expressing L and X in terms of S and w, Problem $\wp^G$ could be written as:

$$\underset{S,w}{\text{Max}} \quad \{\Pi(w, S) - \mu_Z wS - \mu_\varepsilon (1 - w)S + RK)\}$$

subject to

$$K \geq \gamma RWA^G = \frac{VaR_{1-a}(\tilde{Z}w + (1 - w)\tilde{\varepsilon})S - \Pi(w, S)}{R} \tag{15}$$

$$K \geq \chi \left(wS + \alpha(1 - w)S \right) \tag{16}$$

Optimisation problem with constraints applied at the business unit's level   When both constraints are applied at the business unit level, the bank's problem, denoted as $\wp^B$, is as follows:

$$Problem \quad \wp^B : \qquad \underset{L,X}{\text{Max}} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (12), (13) as well as the following two LR requirements

$$K_L \geq \chi L \quad \text{and} \quad K_X \geq \chi \alpha X$$

and the capital allocation constraint

$$K \geq K_L + K_X$$

After reformulating Problem $\wp^B$ in terms of w and S, we get:

$$\underset{S,w}{\text{Max}} \quad \{\Pi(w, S) - \mu_Z wS - \mu_\varepsilon (1 - w)S + RK)\}$$

subject to

$$K_L \geq \gamma RWA^L = \frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w, S)}{R} \tag{17}$$

$$K_X \geq \gamma RWA^X = \frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w, S)}{R} \tag{18}$$

$$K_L \geq \chi w S \tag{19}$$

$$K_X \geq \chi \alpha (1-w) S \tag{20}$$

$$K \geq K_L + K_X \tag{21}$$

4.2   Bank's optimal investments

We are now equipped to compare the bank's investment policy, especially the bank's asset risk, between the two above scenarios. Denote by (w^G, S^G) and (w^B, S^B) the solutions to, respectively, Problem $\wp^G$ and $\wp^B$.

To get first intuitions on how investment decision of the bank differs between the two cases, let us compare constraints of Problem $\wp^G$ to those of Problem $\wp^B$. Clearly, we see that the group-level LR constraint is weakly looser than business unit-level LR constraints. The group-level RW constraint is also looser than the business unit-level RW ones and the gap can be expressed in terms of Div defined as follows:

$$Div = VaR_{1-a}(\tilde{Z}w) + VaR_{1-a}((1-w)\tilde{\varepsilon}) - VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon}) \tag{22}$$

Div represents the diversification benefit per unit of size to the bank if the RW constraint is applied at the consolidated level.

These first observations imply that applying regulatory constraints at the business unit level will reduce the set of investment opportunities available to the bank. The following proposition highlights the efficiency losses resulting from allocating both constraints down to business units.

16

Proposition 1. *Efficiency losses:*

$$S^B \leq S^G$$

Proof. It is the direct consequence of the fact that constraints of Problem $\wp^B$ are weakly tighter than those of Problem $\wp^G$ □

Turning to the impact on the bank's asset risk of allocating constraints down to business units, for the purpose of characterising this impact, it is useful to define the average risk weight (ARW) of the bank's group as

$$ARW^G = \frac{RWA^G}{[w + \alpha(1-w)]S}$$

and of each business unit as

$$ARW^L = \frac{RWA^L}{wS} \quad \text{and} \quad ARW^X = \frac{RWA^X}{\alpha(1-w)S}$$

LR-constrained bank We examine first the case where the bank is bound by the LR requirement at the group level. This happens when Constraint (16) is tighter than Constraint (15), which implies $ARW^G < \frac{\chi}{\gamma}$. We state in the following proposition our first result related to the bank's asset risk.

Proposition 2. *When the bank is bound by the LR requirement at the group level (i.e. $ARW^G < \frac{\chi}{\gamma}$), we have:*

1. $w_B = w_G$ *if*
$$ARW^L \leq \frac{\chi}{\gamma} \quad \text{and} \quad ARW^X \leq \frac{\chi}{\gamma}$$

2. $w_B < w_G$ *if*
$$ARW^L \geq \frac{\chi}{\gamma} \quad \text{and} \quad ARW^X \geq \frac{\chi}{\gamma}$$

Proof. See Appendix. □

17

The first part of Proposition 2 states that if both business units have average risk weight below $\frac{\chi}{\gamma}$, which implies that they will be bound by the LR when the two requirements are allocated down, then the asset risk of the LR-constrained bank does not change with the application level of requirements. That is because in this situation the optimal investment at the group level already allows two business units to comply with the relevant constraint - the LR.

The second part of the proposition specifies that changing the application level from group to business unit will lead to a decrease of the asset risk of the LR-constrained bank if both business units have high average risk weight and thus will be bound by the RW constraint when regulatory requirements are applied at the business-unit level. Two remarks are in order here.

First, this situation where the group is bound by the LR but business units bound by the RW constraint can happen if the diversification benefit is large enough so that all three below inequalities can be satisfied simultaneously:

$$\frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w,S)}{R} > \chi wS \tag{23}$$

$$\frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w,S)}{R} > \chi \alpha (1-w)S \tag{24}$$

and

$$\chi wS + \chi \alpha (1-w)S > \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R} \tag{25}$$

Clearly, the condition for Inequalities (23), 24 and 25 being compatible with each other is as follows:

$$VaR_{1-a}(\tilde{Z}w)S + VaR_{1-a}((1-w)\tilde{\varepsilon})S > VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S \tag{26}$$

or, in words, the diversification benefit Div is high enough.

Second, to get the intuition on why there is a decrease in the bank's asset risk in this case, it is useful to compare the two first order conditions (FOC) that determine w^G and w^B. Indeed, w^G is determined by

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{LR}\left(\chi - \alpha\chi\right) \qquad (27)$$

while w^B is determined by

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] =$$

$$\lambda^L_{VaR}\left[\underbrace{\frac{VaR_{1-a}(\tilde{Z}) - G'(w^B S^B) + R}{R}}_{\frac{\partial(\gamma RWA^L)}{\partial L}} - \underbrace{\frac{VaR_{1-a}(\tilde{\varepsilon}) - (F'((1-w^B)S^B) + c) + R}{R}}_{\frac{\partial(\gamma RWA^X)}{\partial X}}\right] \qquad (28)$$

where λ_{LR} and λ^L_{VaR} are the shadow price of, respectively, the group LR requirement and the business-unit RW requirements.

Equations (27) and (28) equate, on the left hand side (LHS), the marginal benefit of moving one unit of investment from repo business to lending business with its marginal cost on the right hand side (RHS). The former is the increase in the bank's expected marginal profit due to higher profitability of lending business while the latter is measured in terms of marginal changes in required capital resources. Comparing (27) and (28), we see that what drives the difference between w^G and w^B is the marginal cost. When the two constraints are applied at the group level (i.e. Equation (27)), since the binding constraint at this level is the LR, the marginal cost is determined by the difference between the leverage cost per unit of lending (χ) and the leverage cost per unit of repo ($\alpha\chi$). When the two constraints are applied at the business unit level (i.e. Equation (28)), the binding constraints are the RW requirements, which implies that the marginal cost is now determined by the difference between the marginal RW capital cost of lending business and that of repo business. Since repo business has higher leverage cost but lower marginal RW capital cost than lending business, reallocating investment from repo business to lending business will lead to an increase of required capital resources in the case of applying constraints down to business unit level but to a decrease in those resources when constraints are applied at the group level.

Therefore the bank will prefer to invest relatively more in the lending business in the latter case than in the former one. Put differently, the bank's asset risk is lower when regulatory constraints are applied at the business unit level.

RW-constrained bank   We now turn to the case where the RW constraint binds at the group consolidated level. This is equivalent to Constraint (16) being looser than Constraint (15) or $ARW^G > \frac{\chi}{\gamma}$. The following proposition formally states three conditions that will make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are allocated down to business units.

Proposition 3. *When the banks is bound by the RW constraint at the group level (i.e. $ARW^G > \frac{\chi}{\gamma}$), it can happen that $w^B > w^G$ if the following conditions are satisfied:*

1. $ARW^L \geq \frac{\chi}{\gamma}$ *   and   * $ARW^X \leq \frac{\chi}{\gamma}$

2. $\chi\alpha \geq \frac{\partial(\gamma RWA^X)}{\partial X}$

3. $VaR_{1-a}(\tilde{Z}) - VaR_{1-a}(\tilde{\varepsilon}) - \frac{\partial VaR_{1-a}(\tilde{Z}w + \tilde{\varepsilon}(1-w))}{\partial w} < 0$

Proof. See Appendix. □

Similarly to the case of the LR-constrained bank, the impact of allocating down regulatory constraints to business unit on the asset risk of the RW-constrained bank depends on which constraints are binding at the business unit level. Proposition 3 considers the case where, as implied by its first condition, the lending unit is bound by the RW requirement while the repo unit is bound by the LR requirement. In this situation, Proposition 3 indicates that if, following the second condition, the leverage cost per unit of repo is higher than the marginal RW capital cost of this business and if, as stated by the third condition, the diversification benefit Div is decreasing with the share of the lending business in the bank's total balance sheet, then changing the application level from group to business unit can lead to an increase of the asset risk of the RW-constrained bank.

To understand why three conditions specified in Proposition 3 make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are

allocated down to business units, it is again useful to compare the two FOCs that determine w^G and w^B. For a RW-constrained bank, if $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \leq \frac{\chi}{\gamma}$, then w^G and w^B are characterised respectively by

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R = \lambda_{VaR}\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \frac{\partial(\gamma RWA^X)}{\partial X}\right.\right.$$
$$\left.\left. - \frac{VaR_{1-a}(\tilde{Z}) - VaR_{1-a}(\tilde{\varepsilon}) - \frac{\partial VaR_{1-a}(\tilde{Z}w + \tilde{\varepsilon}(1-w^G))}{\partial w}}{R}\right] \quad (29)\right.$$

and

$$\left[G'(w^B S^B) - \mu_Z\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon\right] = \lambda_{VaR}^L \left[\frac{\partial(\gamma RWA^L)}{\partial L} - \chi\alpha\right] \quad (30)$$

for w^B

Equation (29) and (30) also equate the marginal benefit of moving one unit of investment from repo business to lending business with its marginal cost. We can see that if the last two conditions of Proposition 3 are satisfied, then the term in the square bracket on the RHS of Equation (29) is higher than the corresponding term on the RHS of Equation (30). This in turn means that the marginal cost of reallocating investment from repo business to lending business in the case where two constraints apply at the group level can be higher than in the case where constraints are allocated down. If so, the bank will prefer to invest relatively more in the lending business in the latter case than in the former one or, in other words, the bank's asset risk is higher when regulatory constraints are applied at the business unit level. Note that the three conditions stated in Proposition 3 are not sufficient condition for the increase in the bank's asset risk since the RHS of two Equations (29) and (30) also depend on the shadow price of the regulatory constraints.

5 Model calibration

In the previous analysis, we intentionally kept our theoretical setup very general to emphasise the generality of our insights. That generality however implies that we cannot characterise analytically all the possible changes in the bank's investments following the allocation of regulatory constraints to its business units. In this section, to complement those analytical insights with numerical simulations, we calibrate our model to data for banks in the UK. We first make additional assumptions about specific functional forms for the lending and repo incomes to identify further parameters to be calibrated. Then, we describes the data used for the calibration and explain our calibration methods.

Parameters to be calibrated The bank's ex-ante gross interest income from loans $G(L)$ is naturally the product of the loan volume L and the gross interest rate charged on loans. We assume that the interest rate is a decreasing function of the loan volume: $g_1 + g_2 L$ where $g_1 > 0$ and $g_2 < 0$. Therefore, we have:

$$G(L) = (g_1 + g_2 L)L$$

In line with the literature, we also assume that the losses per unit of loans $\tilde{Z}$ are log-normally distributed with parameter μ_Z^{log} and σ_Z^{log}.

In relation to the repo income, we focus here on the role of the repo unit as a market maker in the repo market. Therfore, $F(X)$ is the revenue from reverse repo activities net of the cost of repo activities. We assume that the interest rates charged on both repo and reverse repo depend on the borrowing amount, which implies that

$$F(X) = \underbrace{(d_1 + \varepsilon_1 X)X}_{\text{reverse repo revenue}} - \underbrace{(d_2 + \varepsilon_2 X)X}_{\text{repo cost}}.$$

We further define $\beta_1 = d_1 - d_2$ and $\beta_2 = \varepsilon_1 - \varepsilon_2$ as the spreads between the two transactions, where $\beta_1 > 0$ and $\beta_2 < 0$. Due to limitations on the data as explained below, in this part, we assume that repo is riskless, i.e. repo losses $\tilde{\varepsilon}$ is equal to zero with probability 1. Table

1 summarises the set of parameters that need to be calibrated.

Table 1: Parameters to be calibrated

Parameters	Description
a	VaR confidence level
χ	Leverage requirement
c	Coupon of government bond
R	Bank's borrowing cost
g_1	Marginal return on loan
g_2	Curvature of loan return
μ_Z^{log}	Lognormal parameter of loan losses
σ_Z^{log}	Lognormal parameter of loan losses
β_1	Marginal return on repo
β_2	Curvature of return on repo

Data   In calibrating the model, we use three main data sources. First, we collect information on performance analysis, asset quality and balance sheet for 15 UK banks for which semi-annual data is available in SNL from 2015 to 2018. Our second source of data is the confidential Sterling Money Market Data (SMMD) of the Bank of England. This dataset is daily, transactional-level data. It contains repo and reverse repo transactions with maturity of up to one year that are denominated in Sterling and secured against UK government-issues securities. The repo and reverse repo transactions reported in this dataset cover 95% of total turnover of the market. They are executed by institutions with significant proportion of total activity in the market among which there are 5 UK banks. Finally, the third dataset is daily yield rates for the 15-year UK government bond retrieved from Factset. Table 2 report the variables we use in these datasets for our calibration.

Calibration methods   We set a series of parameters individually. In line with the Basel III risk-weighted capital requirements and leverage ratio requirement, we set the VaR confidence level a to be equal to 0.001 and the minimum leverage ratio χ equal 3%. For the coupon on government bonds, we proxy it by the 15Y UK gilt yield, as the average of daily yields over the entire period. We set the bank's borrowing cost R to be the average cost of funds of all banks in our sample.

Table 2: Data sources

Variable description	Timespan	Frequency	Data source
Gross loans to customers	2015-2018	Semi annual	SNL
Impaired loans	2015-2018	Semi annual	SNL
Net interest margin	2015-2018	Semi annual	SNL
Cost of funds	2015-2018	Semi annual	SNL
Yield 15Y UK gilt	2015-2018	Daily	Factset
Repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Repo interest rate	2017-2019	Daily	SMMD
Reverse repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Reverse repo interest rate	2017-2019	Daily	SMMD

To estimate the distribution parameters μ_Z^{log} and σ_Z^{log} of the random variable $\tilde{Z}$, we proxy its realised value by the amount of impaired loans per unit of total loans. Then we use the maximum likelihood estimation to fit the lognormal distribution of Z with the distribution of impaired loans.

We employ the least square fitting method to derive parameters g_1 and g_2 that underlies the function of gross lending income from net interest margin reported in our datasets. To do so, we first express the net interest margin of bank i at time t - denoted by $IM_{i,t}$ - via g_1 and g_2 as follows:

$$IM_{i,t} = g_1 + g_2 L_{i,t} - Z_{i,t} - R_{i,t}$$

where $L_{i,t}$ is gross loans to customers; $Z_{i,t}$ is the realised impaired loans and $R_{i,t}$ is the cost of funds - all variables are observed in the data. g_1 and g_2 then can be obtained by estimating the following regression equation:

$$y_{i,t} = g_1 + g_2 L_{i,t} + \eta_{i,t}$$

where $y_{i,t} = IM_{i,t} + Z_{i,t} + R_{i,t}$ and $\eta_{i,t}$ is error term. Both coefficients g_1, g_2 derived from the regression are statistically significant at, respectively, 1% and 5% level.

Similarly, to estimate the repo income, we regress the repo and reverse repo interest rate

- denoted by $f_{i,t}^{repo}$ and $f_{i,t}^{reverse}$ respectively - reported for each transaction on the borrowing amount of that transaction using the equations:

$$f_{i,t}^{reverse} = d_1 + \varepsilon_1 X_{i,t}^{reverse} + \nu_{i,t} \quad \text{and} \quad f_{i,t}^{repo} = d_2 + \varepsilon_2 X_{i,t}^{repo} + \upsilon_{i,t}$$

Both regressions give statistically significant coefficients at 1% level. Afterward, we calculate the marginal return on repo β_1 as equal to $d_1 - d_2$ and the curvature of repo return β_2 as $\varepsilon_1 - \varepsilon_2$. Table 3 reports the calibrated value for all parameters.[4]

Table 3: Calibration UK

Description	Parameters	Calibrated Value
VaR confidence level	a	0.001
Leverage requirement	χ	0.03
Coupon on government bond	c	1.0172
Bank's borrowing cost	R	1.0114
Lending unit		
Marginal return on loan	g_1	1.0369
Curvature of loan return	g_2	$-2.22 \cdot 10^{-5}$
Log-normal parameter of Z	μ_Z^{log}	-4.568
(Mean Z)		(0.015)
Log-normal parameter of Z	σ_Z^{log}	0.913
(Standard deviation Z)		(0.018
Repo unit		
Return on reverse repo - repo	β_1	0.000427
Diminishing return parameter	β_2	$-6.943 \cdot 10^{-4}$

In Figure 2 we exemplify the characteristics of our calibrated bank. As seen in the left panel, consistent with Assumption 2, the marginal gross returns on lending are higher compared to the repo returns. They also decrease at a much slower rate compared to the repo ones as the investment size increases. In terms of riskiness, we observe from the right panel that the average risk weights of our calibrated lending business are higher than $\frac{\chi}{\gamma}$ which is equal to 0.35.

---

[4]Values of parameters are reported, when appropriate, in terms of billion GBP.

Figure 2: Bank's characteristics full sample
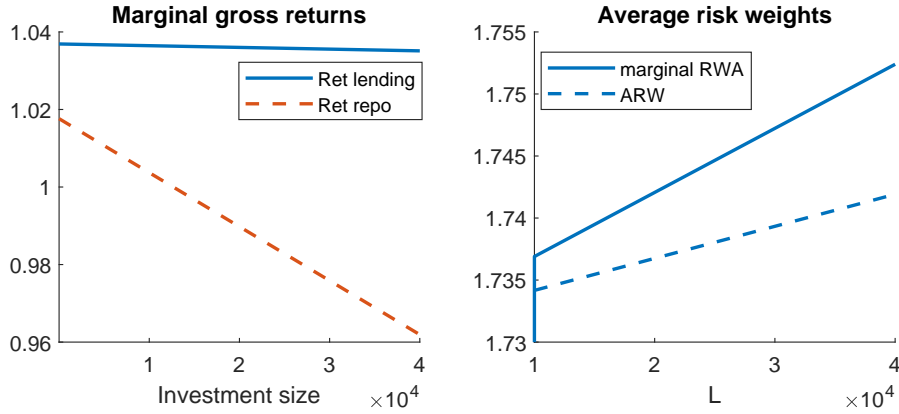
Note: This figure displays main risk and return characteristics of the two business units of our calibrated bank. The left panel shows the marginal gross returns of repo (Ret repo) and lending (Ret lending). The right panel shows the marginal RWA of lending business and its average risk weights of lending (ARW).

6   Numerical simulations

Using the calibrated parameter values, we solve numerically, for different values of the bank's initial equity K, the two optimisation problems $\wp^G$ and $\wp^B$ as defined in Section 4.1. Note that depending on the value of K, the bank can be bound at the group level by either both LR and RW constraints (dark pink area) or only the RW constraint (light blue area). Figure 3 compares the bank's optimal investments between the case where all constraints are applied at the group level and the case in which both business units have to comply with both constraints individually.

We can see that the allocation of constraints leads to efficiency losses since the total investments are reduced (see bottom right panel). As explained in the Section 4, these losses are due to the fact that, when allocating regulatory constraints to its business units, the bank cannot exploit the diversification of its investment portfolio to increase the total size of the portfolio for each level of capital resource.

In term of the impact of the allocation on the bank's asset risk, from the top left panel,

Figure 3: Bank's optimal investments
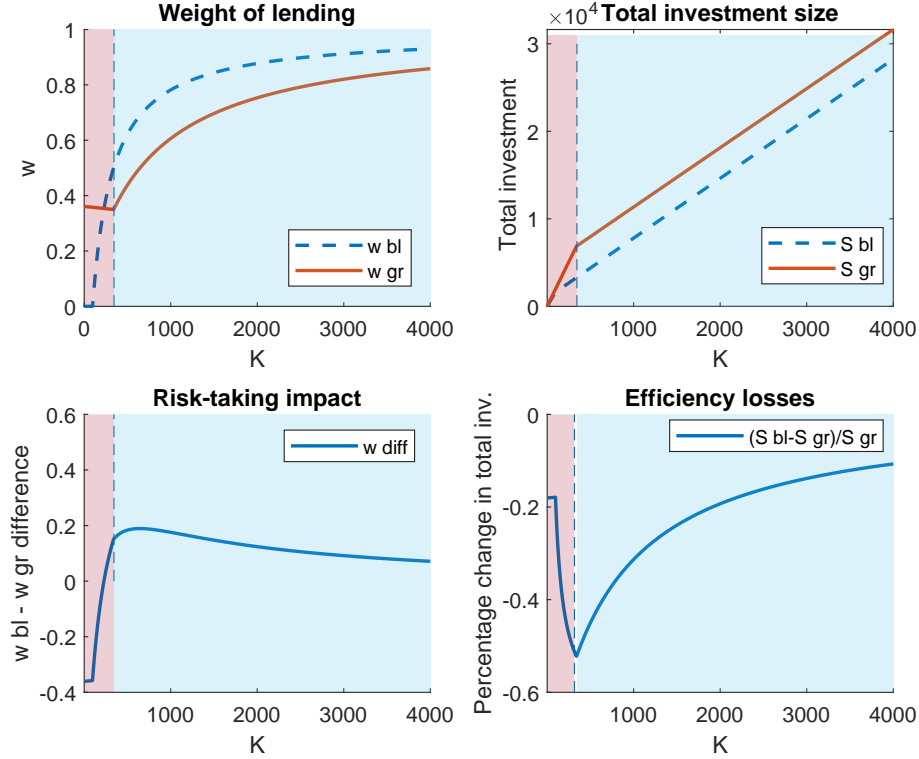
Note: This figure compares the bank's optimal investments in two cases:(i) when both regulatory constraints are applied at the group consolidated level and (ii) when the bank allocates both constraints down to its business units. In the top two panels, the red solid lines represent bank's choices in the first case while the blue dashed lines stand for bank's choices in the second case. The two bottom panels show the difference, between the two cases, in the total size of the bank's balance sheet (bottom right panel) and in the fraction of the balance sheet invested in lending business (bottom left panel). For all panels, the dark pink area corresponds to the situation where both leverage and RW constrains bind at the group level while in the light blue area, only RW constrain matters at the group level.

we can see that this impact depends on whether the bank is constrained only by the RW constrains at the group level (light blue area) or by both RW and LR constraints (dark pink area). When only the RW constraint matters at the consolidated level, requiring all business units to comply with both regulatory constraints will lead to a distortion of investment policy in the sense that the bank will invest relatively more into riskier business - lending. This in turn will increase the overall asset risk of the bank. When both constraints matter at the consolidated level, the impact on the bank's asset risk is somehow ambiguous. When K is very small, the asset risk is decreasing but when K is above some level, the bank's asset risk

increases following the allocation of constraints.

7   Role of business model

As highlighted in the analytical part, the impact of the allocation of constraints on banks'
investment decisions can depend on the diversification benefits, which in turn depends on
the specific characteristics, such as riskiness, of their investments. We therefore expect
that this impact will vary with banks' business model. In this section, we examine this
potential effect of business model. We first classify the 15 UK banks in our SNL dataset into
different business models. Then we recalibrate the lending business for each type and run
the numerical simulations. Note that since the limited number of UK banks in the SMMD
database that we use to calibrate the repo unit does not allow us to have a meaningful
business model classification, we focus here on the consequences of the difference in lending
business characteristics and funding cost between business models.

7.1   Business model classification and calibration

We classify our sample in three types of banks: capital market oriented, wholesale, and
retail-funded banks using a bank business classification methodology proposed in the seminal
paper of Roengpitya et al. (2014). Roengpitya et al. (2014) use a statistical clustering method
based on various ratios of banks' balance sheet which are informative on the bank business
model. They find that retail-funded banks have a high share of gross loans and rely more
on stable sources of funding, such as deposits. The wholesale-funded banks have a lower
percentage of funding coming from deposits, but a higher share of inter-bank liabilities
compared to retail banks. Lastly, the capital markets-orientated banks have a much higher
percentage in trading assets and liabilities compared to the previous two types. The last
type of banks has the highest ratio of inter-bank borrowing as percentage of total assets and
also display a lower reliance on stable funding. The paper reports average values of these
ratios to total assets, and we use them as a benchmark to construct the selection criteria for

our sample.

Due to limited data availability compared to the Roengpitya et al. (2014), we use a restricted version of their selected ratios, and we adjust downwards the threshold criteria to match our sample. Our criteria include: the ratio of customer deposits to total liabilities for the stable source of funding ratio, the ratio of assets held for trading to total assets as a measure of tradable assets, loans to banks as fraction of total assets for our inter-bank lending measure, and bank deposits to total liabilities as the bank deposit ratio. Classifying these ratios based on observed bank characteristics from the Roengpitya et al. (2014), we group our sample into 9 retail-funded, 5 wholesale-funded, and one capital markets-orientated bank. Table 4 reports some characteristics of each business model

Table 4: Business model descriptives

Description	Variable	Values		
		Retail	**Wholesale**	**Capital oriented**
Interest rate unsecured debt	R	0.0129	0.0094	0.0065
		(47)	(27)	(4)
Leverage ratio	LR	0.0549	0.0537	0.0457
		(55)	(39)	(4)
Fully loaded risk weighted		0.2528	0.184	0.187
capital ratio		(43)	(24)	(8)
Loans to total assets	L/total assets	0.7649	0.6601	0.3653
		(63)	(37)	(6)
Percentage of impaired loans		1.11%	1.99%	2.74%
to total loan size		(53)	(26)	(2)

The number of observations is in brackets, unless otherwise stated.

We recalibrate the model to the different categories of banks. In this step, we combine the last two groups and split our sample in 'Retail banks' and 'Wholesale and capital markets orientated' since with only one capital market-oriented banks, the regression analysis would not have enough observations to be reliable. We report the calibrated values of different parameters for each types of banks in Table 5.

Comparing the borrowing cost R between retail banks, wholesale and capital markets-oriented banks and our full sample, we see that, on average, the capital markets-oriented

Table 5: Calibration UK business models

Description	Parameters	**Retail banks**	**Wholesale and cap.**
Bank's borrowing cost	R	0.0129	0.009
Lending			
Marginal return on loan	g_1	1.0369	1.03081
Curvature of loan return	g_2	$-3.15 \cdot 10^{-5}$	$-1.03 \cdot 10^{-5}$
Log-normal parameter of Z (Mean Z)	μ_Z^{log}	-4.885 (0.0118)	-3.97 (0.0207)
Log-normal parameter of Z (Standard deviation Z)	σ_Z^{log}	0.945 (0.0142)	0.429 (0.0093)

banks have the lowest cost of funding while retail banks have the highest cost. To compare the risk characteristics between these categories, we present the distribution of impaired loans and the diversification benefits[5] in Figure 4. We observe that wholesale and capital markets-oriented banks have lowest level of risk in lending business and highest diversification benefits in their investment portfolio.

Further, in Figure 5, we compare the two business models in terms of returns and risk weights. The marginal returns of lending are higher for the retail bank group, and decrease at a higher speed compared to the wholesale and capital market-oriented banks as seen in top left panel. The retail group has a higher variation in losses compared to second group, and that can be seen via a higher value of Risk Weighted Assets in the bottom left panel. The ARWs for retail are larger in absolute terms compared to the wholesale and capital-markets oriented group (dotted blue line from top right panel compared to dotted orange line from bottom right panel), but they are both above the threshold $\frac{x}{\gamma} = 35\%$. The difference between the two groups indicates that lending is a more profitable investment relatively to repo for the retail banks compared to the second group, aspect which is later confirmed in the optimal

---

[5]Note that since in the numerical simulation, we assume that repo activities are riskless, the diversification benefit is therefore computed as $RWA^G - RWA^L$

Figure 4: Risk characteristics and diversification benefits across business models
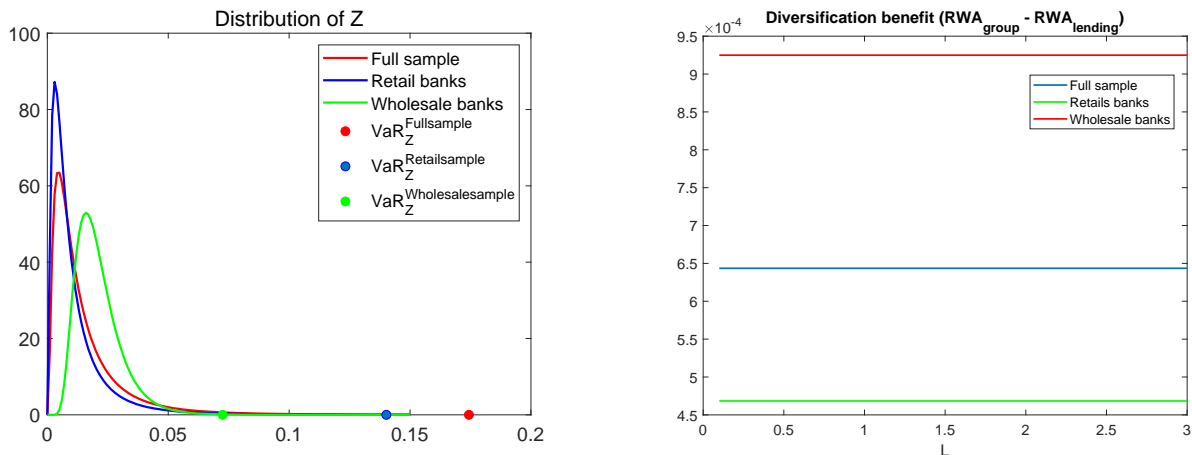
Note: The left panel displays the distributions of non-performing loans to total loans (Z) based on the calibrated values for the full sample, retail banks and wholesale and capital oriented banks (denoted by wholesale in graph). The right panel shows the diversification benefits which are computed, since repo is assumed to be riskless, as the RWA at group level minus the RWAs of lending.

investment solutions. Moreover, given the stronger diminishing marginal returns in the retail group, we expect the overall retail bank size to be smaller compared to the wholesale and capital bank size for equal initial capital.

7.2   Numerical simulations for different business models

We now run the simulations for each of the two types of banks. Figure 6 compares the optimal investments of both retail and wholesale and capital markets-oriented banks between the case where all constraints are applied at the group level and the case in which the bank chooses to allocate both constraints to its business units.

Three main observations are in order here. First the situation in which the leverage constraint binds at the group consolidated level happens only with wholesale and capital market-oriented banks but not for retail banks. Note also that since in this simulation, we assume that repo business is riskless, the average risk weight of repo business for both types of banks is lower than 35%. This difference can therefore be explained by the fact that the average risk weight of the lending business for both types of banks is higher than $\frac{\chi}{\gamma} = 35\%$

Figure 5: Bank's characteristics business model comparison
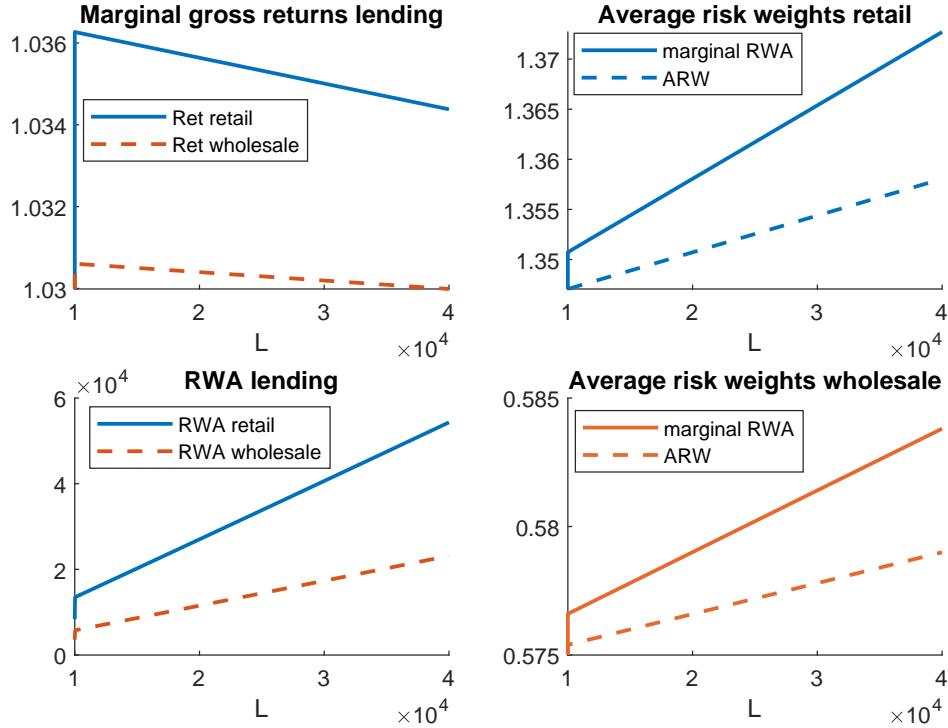
Note: This figure compares main characteristics of banks across business models. The top left panel captures the marginal gross returns of lending, where the blue filled line is the marginal return of the retail group (Ret retail), and the orange dotted one of the wholesale and capital orientated group (Ret wholesale). The bottom left panel captures the total value of risk weighted assets of lending for the two groups as a function of lending. In the two right panels we exemplify the average risk weights (ARW) and the marginal RWA as a function of investment in lending for retail, and wholesale and capital oriented banks respectively.

but the diversification benefits of the wholesale banks are higher than retail banks as shown in Figure 4.

Second, there is a stark difference in the impact of the allocation of constraints on the banks' asset risk between retail type and wholesale and capital markets-oriented type in the case where only the RW constraint binds at the group level (beige area in Figure 6). Precisely, in this case, while the allocation of constraints results in an increase in asset risk of retail banks, it brings about a decrease in asset risk of wholesale banks.

Finally, when comparing the simulation results of each type of banks with the ones of our

full sample, we could see that the average bank in our full sample behave in a very similar way with retail banks.

Figure 6: Optimal investment comparison
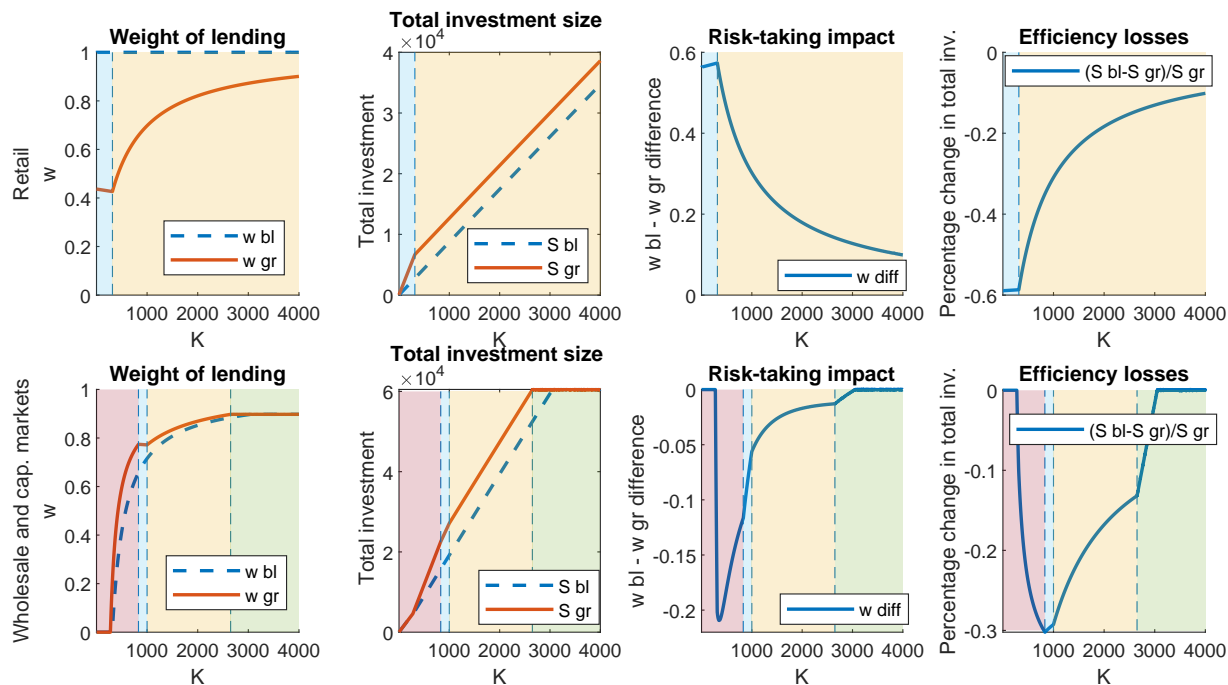
Note: This figure compares the optimal investments of retail and wholesale and capital markets-oriented banks in two cases:(i) when both regulatory constraints are applied at the group consolidated level and (ii) when the bank allocates both constraints to its business units. In the 4 top panels, the red solid lines represent bank's choices in the first case while the blue dashed lines stand for bank's choices in the second case. The first row of panels shows the retail banks group results, while the second row displays the results of the wholesale and capital orientated markets bank group. The panels in the third and 4th column represent the difference in the total size of the bank's balance sheet (forth top and bottom panels) and in the fraction of the balance sheet invested in lending business (third top and bottom panel) between the two cases. For all panels, the dark pink area corresponds to the situation where the leverage constraint binds at the group level; the light blue area to the case in which both leverage and RW constrains bind; the beige are to the case only RW constraint matters and finally the green area is when no constraints bind. Notice how for retail we have only the case when either both RW constraint and LR bind at group level, or when only the RW constraint binds.

8   Conclusion and policy implications

In this paper we evaluate the risk-taking implications of introducing the leverage ratio requirement. More precisely, we assess how banks' asset risk depends on whether the bank applies the leverage ratio requirement at the consolidated level alone, or at each individual business line. We develop a two-period theoretical model where the bank has two business units defined by two investment opportunities: a riskier one which yields a higher margin, and a less risky one which has low returns. We refer to these units as lending and repo business units, respectively. We calibrate the model to UK banks, and further evaluate and how do these risk-taking implications depend on the business model of the banks.

We find that the effects on asset risk broadly depend on the most binding constraint at the banks' group level, and on the relationship between the average risk weights of each unit and the ratio of the minimum requirements for leverage ratio and the risk weighted capital requirement. Firstly, when requirements are applied exclusively at the consolidated level, the bank enjoys diversification benefits as the two units can complement each other: the leverage ratio has a relatively higher impact on the capital requirements of the repo unit, while the risk-weighted asset requirement penalizes more the lending unit. Hence, allocating constraints at business units can lead to a loss of the diversification benefit. We find that this leads to either the same, or a decrease in total investment.

In terms of most binding constraints, when the bank is bounded by the leverage ratio at the group level, we find that allocating requirements to business units leads to no increase in risk-taking. As long as both business units have average risk weights below a certain threshold on a stand-alone basis, then optimal investment is not affected by the level at which the regulatory requirements are applied. If the ARW are higher than the threshold, then requiring each of the business units to comply to the leverage ratio requirement will lead to a decrease of investment in the lending unit. In the case when the bank is bounded by the RW requirement at a group level, allocating constraints to business units leads to a distorted investment decision. If allocating constraints leads to a RW constrained lending unit, and a LR constrained repo unit, and the bank has decreasing diversification benefits

from applying the risk-weighted capital requirement at the consolidated level, then allocating requirements at business units most likely leads to an increase in the relative investment in lending as compared to the total investment.

We capture additional insights when we calibrate the model to UK banks in terms of efficiency losses and the impact on asset risk of allocating constraints to business units. If only the risk-weighted requirement matters at the consolidated level, we find that the investment is distorted, leading to a higher relative investment in the riskier unit. Both the efficiency loss and risk-taking effects diminish as the overall investment size increases. If the bank is bounded by the LR requirement and the RW capital requirement simultaneously, the results are ambiguous in terms of asset risk. Lastly, when we calibrate the model based on different business models, we find that applying constraints at business units leads to an increase in risk-taking for retail banks, while we obtain the opposite effect for wholesale and capital-orientated banks. These results depend on the different factors explained above.

From a policy perspective, our paper generates useful insights on the current debate on what is the appropriate level of application of the leverage ratio requirement. Firstly, we highlight two potential costs when applying this requirement at the business unit level: it can induce banks to increase their asset risk and it can make banks decrease their overall investment, generating inefficiency losses. This has immediate implications for low-risk, low-margin markets, as their size may decrease in favour of riskier investments with a higher expected non-risk adjusted return. Applying the leverage ratio requirement at business unit level has the potentiality of systemic risk build-up, and policy makers and supervisors should closely monitor both the levels at which these requirements are being applied, and the market liquidity in low-risk activities such as the repo market. Secondly, we find that one size does not fit all, and the impact on asset risk differs across bank types, depending on which regulatory requirement (RW or LR) matters most for the bank at group level. Hence we argue that bank supervision should be tailored to take into account bank business models which in turn relates to the most binding constraint, as it otherwise leads to distorted asset risk decisions.

In further work we aim to expand the model to cross-lending, where one of the two business units funds the other. The cash generated in the repo transaction can be used as a source of cheap funding for the banks' lending unit. Moreover, at the moment we analyse the extreme case when banks cascade down completely their requirements at business line levels. What we observe in practice is that banks also attach specific weights to the two requirements when they apply the constraints at business unit levels, with the highest weight given to the most binding constraint. We aim to further enhance this research work by analysing the effects of attaching different weights to each of the two requirements.

References

Acosta-Smith, J., Grill, M., and Lang, J. H. (2018). The leverage ratio, risk-taking and bank stability. *Bank of England Staff Working Paper No. 766*.

Allahrakha, M., Cetina, J., and Munyan, B. (2018). Do higher capital standards always reduce bank risk? the impact of the basel leverage ratio on the us triparty repo market. *Journal of Financial Intermediation*, 34:3–16.

Bajaj, R., Binmore, A., Dasgupta, R., and Vo, Q.-A. (2018). Banks internal capital markets: how do banks allocate capital internally? *Bank of England Quarterly Bulletin Q2*.

Baker, M. and Wurgler, J. (2015). Do strict capital requirements raise the cost of capital? bank regulation, capital structure, and the low-risk anomaly. *American Economic Review*, 105(5):315–20.

Barth, A. and Seckinger, C. (2018). Capital regulation with heterogeneous banks–unintended consequences of a too strict leverage ratio. *Journal of Banking & Finance*, 88:455–465.

Basel Committee (2009). Strengthening the resilience of the banking sector. *Basel Committee*.

Bicu-Lieb, A., Chen, L., and Elliott, D. (2020). The leverage ratio and liquidity in the gilt and gilt repo markets. *Journal of Financial Markets*, page 100510.

BIS CGFS (2017). Repo market functioning report. *Committee on the Global Financial System Papers*, no 59.

Blum, J. M. (2008). Why Basel II may need a leverage ratio restriction. *Journal of Banking & Finance*, 32(8):1699–1707.

Choi, D. B., Holcomb, M. R., and Morgan, D. P. (2018). Leverage limits and bank risk: New evidence on an old question. Technical Report No. 856, Staff Report Federal Reserve Bank of New York, New York NY.

Dhaene, J., Tsanakas, A., Valdez, E. A., and Vanduffel, S. (2012). Optimal capital allocation principles. *Journal of Risk and Insurance*, 79(1):1–28.

Duffie, D. (2018). Financial regulatory reform after the crisis: An assessment. *Management Science*, 64(10):4835–4857.

Gambacorta, L. and Karmakarb, S. (2018). Leverage and risk-weighted capital requirements. *International Journal of Central Banking*.

Goel, T., Lewrick, U., and Tarashev, N. (2019). Bank capital allocation under multiple constraints. *Journal of Financial Intermediation*, page 100844.

Graham, J. R. and Harvey, C. R. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics*, 60(2-3):187–243.

Ita, A. (2017). Capital allocation in large banks-a renewed look at practice. *Available at SSRN 2726165*.

Khaykin, I., Koyluoglu, U., Elliott, D., and Spicer, C. (2017). Balancing complex and competing constraints on capital, liquidity and funding. *Oliver Wyman. Financial Resource Management. Technical report.*

Kiema, I. and Jokivuolle, E. (2014). Does a leverage ratio requirement increase bank stability? *Journal of Banking & Finance*, 39:240–254.

Kotidis, A. and Van Horen, N. (2019). Repo market functioning: The role of capital regulation. *CEPR Discussion Paper No. DP13090*.

Krüger, P., Landier, A., and Thesmar, D. (2015). The WACC fallacy: The real effects of using a unique discount rate. *The Journal of Finance*, 70(3):1253–1285.

Perold, A. F. (2005). Capital allocation in financial firms. *Journal of Applied Corporate Finance*, 17(3):110–118.

Roengpitya, R., Tarashev, N. A., and Tsatsaronis, K. (2014). Bank business models. *BIS Quarterly Review December.*

Stein, J. C. (1997). Internal capital markets and the competition for corporate resources. *The Journal of Finance*, 52(1):111–133.

A   Appendix

A.1   Derivation of Constraint (11) - the RW constraint at the group level

Given that

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L) \tag{31}$$

and

$$\tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X) \tag{32}$$

we can write $\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a$ as

$$\mathbb{P}\left(G(L) + F(X) + cX - R(X + L - K) \leq \tilde{Z}L + \tilde{\varepsilon}X\right) \leq a \tag{33}$$

Using Definition (6), Inequality (33) is equivalent to

$$VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) \leq G(L) + F(X) + cX - R(X + L - K)$$

or

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - [G(L) - RL + F(X) + cX - RX]}{R} \tag{34}$$

A.2   Proof of Proposition 2

- *First, we prove that if both business units have average risk weight below $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank does not change with the application level of requirements.*

To prove the above, we will establish that the solution (w^G, S^G) to Problem $\wp^G$ will also be the solution to Problem $\wp^B$ if the three following conditions are satisfied:

$$\chi(wS + \alpha(1-w)S) > \gamma RWA^G \tag{35}$$

as well as

$$\frac{RWA^L}{wS} \leq \frac{\chi}{\gamma} \quad \text{and} \quad \frac{RWA^X}{\alpha(1-w)S} \leq \frac{\chi}{\gamma} \tag{36}$$

Indeed, since (w^G, S^G) is the solution to Problem $\wp^G$ when Condition (35) is satisfied, we have:

$$K = \chi(w^G S^G + \alpha(1-w^G)S^G) \tag{37}$$

When two conditions in (36) hold, the relevant constraints for Problem $\wp^B$ will be Constraints (19), (20) and (21). Clearly, (w^G, S^G) that satisfies Equality (37) will also satisfy all Constraints (19), (20) and (21) where we simply choose $K_L = \chi w^G S^G$ and $K_X = \chi \alpha (1-w^G)S^G$. This in turn implies that (w^G, S^G) belong to the feasible set of Problem $\wp^B$. Since the feasible set of Problem $\wp^B$ is smaller than that of Problem $\wp^G$, (w^G, S^G) are also the solution to Problem $\wp^B$.

- *We now prove that if both business units have average risk weight greater than $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank will decrease when regulatory constraints are allocated down to business units.*

We first derive the two FOCs that determine w^G and w^B. The Lagrangian for Problem $\wp^G$ reads:

$$\Lambda_g = \Pi(S, w) - \mu_Z wS - \mu_\varepsilon (1-w)S + RK + \lambda_{VaR}\left(K - \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R}\right)$$
$$+ \lambda_{LR}(K - \chi(w + \alpha(1-w))S)$$

where λ_{VaR} and λ_{LR} are the Lagrange multiplier for, respectively, the RW constraint and the LR constraint.

The FOC that determines w is as follows:

$$\frac{\partial \Lambda_g}{\partial w} = \frac{\partial \Pi(S, w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \frac{S \lambda_{VaR}}{R} \frac{\partial VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})}{\partial w} +$$
$$+ \frac{\lambda_{VaR}}{R} \frac{\partial \Pi(w, S)}{\partial w} - \lambda_{LR} \chi(1-\alpha)S = 0$$

When the LR requirement is the binding constraint at the group level, we have $\lambda_{VaR} = 0$ and $\lambda_{LR} \geq 0$. Therefore, w^G is determined by the following equation

$$\left[G'(w^G S^G) - \mu_Z - R \right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R \right] = \lambda_{LR} \left(\chi - \alpha\chi \right) \qquad (38)$$

Similarly, the Lagrangian for Problem $\wp^B$ reads:

$$\Lambda_b = \Pi(S, w) - \mu_Z w S - \mu_\varepsilon (1-w)S + RK + \lambda_{VaR}^L \left(K_L - \frac{VaR_{1-a}(\tilde{Z}w)S - G(wS) + RwS}{R} \right)$$
$$+ \lambda_{VaR}^X \left(K_X - \frac{VaR_{1-a}(\tilde{\varepsilon}(1-w))S - F\big((1-w)S\big) - c(1-w)S + R(1-w)S}{R} \right)$$
$$+ \lambda_{LR}^L(K_L - \chi w S) + \lambda_{LR}^X(K_X - \chi\alpha(1-w)S) + \lambda_K(K - K_L - K_X)$$

where $\lambda_{VaR}^L, \lambda_{VaR}^X, \lambda_{LR}^L, \lambda_{LR}^X$ and λ_K are the Lagrange multipliers of corresponding constraints.

The FOC for w is written as follows:

$$\frac{\partial \Lambda_b}{\partial w} = \frac{\partial \Pi(S, w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \frac{S \lambda_{VaR}^L}{R} \frac{\partial VaR_{1-a}(\tilde{Z}w)}{\partial w} + \frac{\lambda_{VaR}^L}{R} \left(\frac{\partial G(wS)}{\partial w} - RS \right)$$
$$- \frac{S \lambda_{VaR}^X}{R} \frac{\partial VaR_{1-a}(\tilde{\varepsilon}(1-w))}{\partial w} + \frac{\lambda_{VaR}^X}{R} \left(\frac{\partial F((1-w)S)}{\partial w} - Cs + RS \right)$$
$$- \lambda_{LR}^L \chi S + \lambda_{LR}^X \chi\alpha S = 0$$

When $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \geq \frac{\chi}{\gamma}$, Constraint (17) is tighter than Constraint (19) and Constraint (18) is tighter than Constraint (20). Therefore the binding constraints will be Constraints (17) and (18), which imply

$$\begin{cases} \lambda_{LR}^{L} = 0; & \lambda_{LR}^{X} = 0 \\ \lambda_{VaR}^{L} \geq 0; & \lambda_{VaR}^{X} \geq 0 \end{cases}$$

From the two FOCs for K_L and K_X, we have

$$\lambda_{VaR}^{L} + \lambda_{LR}^{L} - \lambda_K = 0 \quad \text{and} \quad \lambda_{VaR}^{X} + \lambda_{LR}^{X} - \lambda_K = 0 \tag{39}$$

Therefore, we obtain

$$\begin{cases} \lambda_{LR}^{L} = 0; & \lambda_{LR}^{X} = 0 \\ \lambda_{VaR}^{L} = & \lambda_{VaR}^{X} \geq 0 \end{cases} \tag{40}$$

Plugging Result (40) into the above FOC, we get

$$\left[G'(w^B S^B) - \mu_Z - R \right] - \left[F'((1 - w^B)S^B) + c - \mu_\varepsilon - R \right] =$$
$$\lambda_{VaR}^{L} \left[\frac{VaR_{1-a}(\tilde{Z}) - G'(w^B S^B) + R}{R} - \frac{VaR_{1-a}(\tilde{\varepsilon}) - (F'((1 - w^B)S^B) + c) + R}{R} \right] \tag{41}$$

Note that the LHS of Equations (38) and (41) is a decreasing function of w. Moreover the RHS of Equation (38) is non positive while the RHS of Equation (41) is non negative. These all together imply that $w^B < w^G$

43

A.3 Proof of proposition 3

Let w^G, S^G be the solution at the group level. In this case, in terms of constraints at optimum:

$$K = \frac{S^G VaR_{1-a}(\tilde{Z}w^G + (1-w^G)\tilde{\varepsilon}) - \Pi(w^G, S^G)}{R}$$

$$K > \chi(w^G + \alpha(1-w^G))S^G$$

Remark 1. w^G, S^G can **not** be a solution to the problem at business unit level since w^G, S^G cannot satisfy VaR constraints at business level.

Proof. Indeed, since

$$\frac{S^G VaR_{1-a}(\tilde{Z}w^G + (1-w^G)\tilde{\varepsilon}) - \Pi(w^G, S^G)}{R} < \frac{S^G VaR_{1-a}(\tilde{Z}w^G) - (G(w^G S^G) - Rw^G S^G)}{R}$$
$$+ \frac{S^G VaR_{1-a}((1-w^G)\tilde{\varepsilon}) - \left[F((1-w^G)S^G) + c(1-w^G)S^G - R(1-w^G)S^G\right]}{R}$$

This inequality implies that $K \leq K_L + K_X$: contradiction with the condition that $K \geq K_L + K_X$.

Remark 2. In the case where banks are bounded by VaR constraint at the group level, when constraints are cascaded down, it can **not** happen that:

$$\begin{cases} \text{RHS of (20)} \geq \text{RHS of (18)} \\ \text{RHS of (19)} \geq \text{RHS of (17)} \end{cases}$$

Remark 3. For the optimization problem at the business unit level, it can **not** happen that one business is bounded by some constraint while the other is not bounded by any constraint.

Remarks 1,2 and 3 imply that when the bank is bounded by VaR constraint at the group

level, when the constraint is cascaded down, there are 3 possible cases:

$$A. \begin{cases} \text{only VaR is binding for lending} \\ \text{only VaR is binding for Repo} \end{cases} \iff \begin{cases} \lambda^L_{VaR} = \lambda^X_{VaR} > 0 \\ \lambda^L_{LR} = 0; \lambda^X_{LR} = 0 \end{cases} \tag{42}$$

$$B. \begin{cases} \text{only VaR is binding for lending} \\ \text{only LR is binding for Repo} \end{cases} \iff \begin{cases} \lambda^L_{LR} = \lambda^X_{VaR} = 0 \\ \lambda^L_{VaR} = \lambda^X_{LR} \geq 0 \end{cases} \tag{43}$$

$$C. \text{ All 4 constraints are binding} \tag{44}$$

We will consider case B. At the business unit level, (w^B, S^B) is defined by :

$$G'(w^B S^B) + F'((1-w^B)S^B)(1-w^B) + c(1-w^B) - R =$$
$$\mu_Z w^B + \mu_\varepsilon(1-w^B) + \frac{\lambda^L_{VaR}}{R}\left(VaR_{1-a}(\tilde{Z}w^B) - G'(w^B S^B)w^B + Rw^B\right) \tag{45}$$
$$G'(w^B S^B) - F'((1-w^B)S^B) - c = \mu_Z - \mu_\varepsilon + \frac{\lambda^L_{VaR}}{R}\left(VaR_{1-a}(\tilde{Z} - G'(w^B S^B) + R) - \lambda^L_{VaR}\chi\alpha\right. \tag{46}$$

At the group level, the solution w^G, S^G is defined by:

$$G'(w^G S^G)w^G + F'((1-w^G)S^G)(1-w^G) + c(1-w^G) - R =$$
$$\mu_Z w^G + \mu_\varepsilon(1-w^G) + \frac{\lambda_{VaR}}{R}\left(VaR_{1-a}(\tilde{Z}w^G + (1-w^G)\tilde{\varepsilon} - G'(w^G S^G)w^G + Rw^G\right) \tag{47}$$
$$G'(w^G S^G) - F'((1-w^G)S^G) - c =$$
$$\mu_Z - \mu_\varepsilon + \frac{\lambda_{VaR}}{R}\left(\frac{\partial VaR_{1-a}(\tilde{Z}w^G + (1-w^G)\tilde{\varepsilon})}{\partial w} - G'(w^G S^G) + R\right)$$
$$+ \frac{\lambda_{VaR}}{R}\left(F'((1-w^G)S^G) + c - R\right) \tag{48}$$

The second equation from (46) can be written as:

$$G'(w^B S^B) - F'((1 - w^B)S^B) - c = \mu_Z - \mu_\varepsilon + \lambda^L_{VaR} \underbrace{\left(\frac{VaR_{1-a}(\tilde{Z} - G'(w^B S^B) + R}{R}\right)}_{\text{marginal RWA L}} - \lambda^L_{VaR}\chi\alpha$$

(49)

The second equation from (48) can be written as:

$$G'(w^G S^G) - F'((1 - w^G)S^G) - c = \mu_Z - \mu_\varepsilon + \lambda^L_{VaR} \underbrace{\left(\frac{VaR_{1-a}(\tilde{Z} - G'(w^G S^G) + R}{R}\right)}_{\text{marginal RWA L}} \quad (50)$$

$$- \lambda_{VaR} \underbrace{\frac{VaR_{1-a}(\tilde{\varepsilon}) - F'((1 - w^G)S^G) - c + R}{R}}_{\text{marginal RWA X}} \quad (51)$$

$$- \frac{\lambda_{VaR}}{R} \underbrace{\left(-\frac{\partial VaR_{1-a}(\tilde{Z}w^G + (1 - w^G)\tilde{\varepsilon}}{\partial w} + VaR_{1-a}(\tilde{Z} - VaR_{1-a}(\tilde{\varepsilon}))\right)}_{\text{marginal diversification benefit}} \quad (52)$$

Therefore, if the three conditions stated in Proposition 3 are satisfied, it can happen that the RHS of Equation (49) is lower than that of Equation (50). $\square$